Appendix B: A discussion on Legendre Transform

$U(S,V)$: if it is known, thermodynamics of system is known

- Introduce $F = U - TS$ (1)

  if $F(T,V)$ is known, thermodynamics of system is known.

- Step (1) is a Legendre Transform
- What is being done when we go from $U(S)$ to $F(T)$?

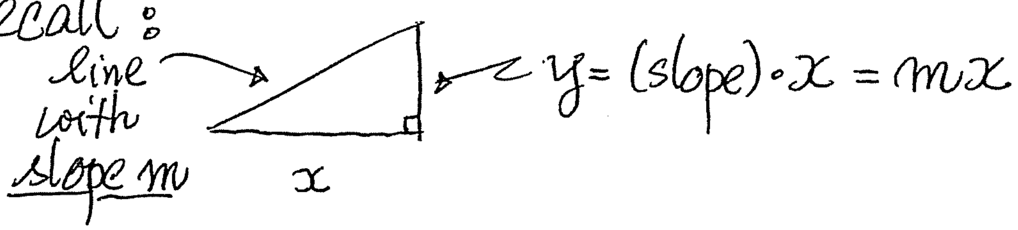
— x —

- Recall: $dU = TdS - pdV$ | From $U(S,V)$ to $F(T,V)$: Easier to think of it as $U(S)$, as V is still there

  $$\Rightarrow \underbrace{\left(\frac{\partial U}{\partial S}\right)_V = T}$$

$$\boxed{\text{Given } U(S)\text{, slope } \frac{\partial U}{\partial S} \text{ is the Temperature}}$$

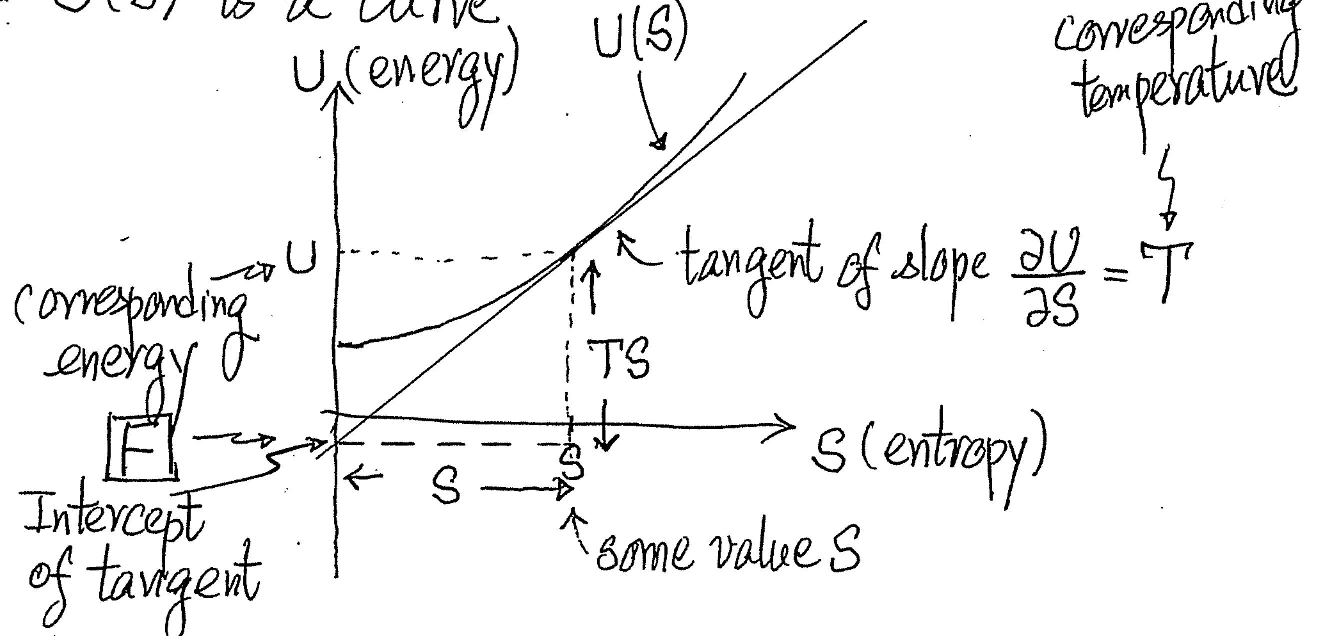
- Recall:



This leads to a geometric interpretation of Legendre Transform

- Just amounts to looking at a painting in different ways!

- $U(S)$ is a curve,



- This point is $U - TS = F$

$$\boxed{\begin{array}{l}\therefore T \text{ is the slope of tangent at a given } S \\ F \text{ is the intercept of the tangent}\end{array}}$$

The point is:

- Two descriptions of the same information
- Given $U(S)$, construct $F(T)$

  (F: intercept; T: slope of $U(S)$)

- Given $F(T)$, can re-construct $U(S)$

Another example in physics

- Lagrangian $L(q,\dot{q})$ focus on $L(\dot{q})$. Slope $\frac{\partial L}{\partial \dot{q}} = p$ (momentum)

Construct: $p\dot{q} - L = H(q,p)$ (Hamiltonian)